JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 30, No. 4, July–August 2007

Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Guidance Strategy for Hyperbolic Rendezvous

Damon F. Landau* and James M. Longuski†
Purdue University, West Lafayette, Indiana 47907-2023

DOI: 10.2514/1.30071

I. Introduction

CYCLER and semicycler architectures are two promising options to sustain the human exploration of Mars. In these architectures, the interplanetary transfer vehicle receives a gravity assist from Earth, Mars, or both planets that sends the vehicle and crew to the next planet. However, the crew must still travel from the surface of Earth or Mars to the transfer vehicle before it can transport them to Mars or Earth. This planetocentric transfer is accomplished by a small "taxi" vehicle, which only supports the crew for a few days. Consequently, the taxi can be significantly less massive than the transfer vehicle, which sustains the crew for many months. The main disadvantage of cycler and semicycler architectures is that the taxi must rendezvous with the transfer vehicle while they are on a hyperbolic trajectory during planetary flyby. If this rendezvous fails, the crew will be lost. (We note that the Apollo missions included a similar risk when the lunar module docked with the command/service module in lunar orbit. If this rendezvous failed, two of the three astronauts would not make it home [1].) Therefore additional safety features may be required for hyperbolic rendezvous to be practical.

II. Rendezvous Strategy

We examine the rendezvous process from a low-circular orbit departure to final docking of the taxi with the cycler. The taxi consists of four stages: 1) the first upper stage, which achieves a high-energy orbit, but does not escape, 2) the second upper stage, which propels the taxi to the same V_∞ as the cycler, 3) the rendezvous engine, which cancels most of the relative velocity between the taxi and cycler, and 4) the docking engines, which achieve final mating of the two vehicles. The first stage burns out before escape so the crew can abort to the surface if there is a problem. The second stage completes the escape ΔV and is smaller than the first stage (which is dropped) to provide redundancy during rendezvous without adding prohibitive mass. The rendezvous stage performs a significantly smaller ΔV than either upper stage and is critical to crew survival. Finally the docking stage requires significantly smaller thrust than the other stages and is similar to a reaction control system. Two engines are used for the first three stages so that the second engine can complete the maneuver if the first engine fails. A redundant docking propulsion system (engine and propellant) is also employed. Because the rendezvous burn is critical to crew survival (without it the taxi will not reach the cycler at a safe speed) we keep the spent second upper-stage engine (which already worked to achieve V_∞) as a backup in case both rendezvous engines fail. Moreover, extra rendezvous propellant could be kept in this stage should the primary rendezvous propellant be lost. Because this measure increases mass significantly we recommend that it is incorporated only in missions that employ in situ propellant production (e.g., creating hydrogen and oxygen from Mars water, or methane and oxygen from Mars CO_2 and hydrogen feedstock). A 30-minute window is provided to switch rendezvous engines before the taxi passes the cycler should the need arise. We assume that the cycler is an entirely passive vehicle during rendezvous. Contingencies for various failure modes are provided in Table 1. The goal is to develop a rendezvous strategy where failure of a system does not result in loss of the crew.

III. Rendezvous Procedure

The taxi and cycler trajectories are modeled as conics in a point-mass gravity field. The maneuvers are modeled as finite burns with varying mass based on the propellant flow rate. We assume constant thrust magnitude and constant direction for all maneuvers and that the engines can be turned on or off at any time and for multiple cycles. (The capability of throttling or vectoring the thrusters is not assumed.) The optimized ΔV (not including the safety features in Table 1) for free departure time, orbit orientation, and thrust duration and direction is about 60 m/s, or 1%, above the impulsive ΔV case. Thrust duration for departure is several minutes and the rendezvous burn is typically less than 1 min.

The taxi employs three maneuvers to reach the docking corridor. The first two are carried out back to back by the first and second upper stages. The first stage achieves a parking orbit with the same period as the expected rendezvous time, so the crew can abort to the surface if the second stage fails. The thrust direction is the same for both burns so the taxi does not have to reorient itself for the second burn. The third maneuver is the rendezvous burn to direct the taxi to the docking corridor. The maneuvers are designed to maximize the final taxi mass with the constraint that the taxi must approach the cycler at a desired direction and speed. We set no constraints on the orientation of the low-circular orbit. (A combined optimization of the launch vehicle and upper-stage capabilities could determine the orbit parameters to maximize final mass.) Because the time between maneuvers is on the order of hours, the desired burn time and direction may be updated during the mission using the current estimated taxi position and velocity. A rendezvous time line is provided in Fig. 1.

The guidance solution (direction and duration of thrust for each maneuver) to approach the docking corridor is essentially a set of constrained optimization problems. We first optimize the parking-orbit orientation and departure time, first and second stage thrust direction and durations, and rendezvous-stage direction and duration, with the constraint that the taxi must be 10 km from the cycler along its sun line and traveling at 40 m/s toward the cycler after some set rendezvous time (say 1 day). The objective function is to maximize the final taxi mass, which is calculated based on the

Presented as Paper 6299 at the AIAA/AAS Astrodynamics Specialist Conference, Keystone, CO, 21–24 August 2006; received 27 January 2007; revision received 6 February 2007; accepted for publication 13 March 2007.

*Doctoral Candidate, School of Aeronautics and Astronautics, 315 North Grant Street. Student Member AIAA.

†Professor, School of Aeronautics and Astronautics, 315 North Grant Street. Associate Fellow AIAA.

Table 1 Failure modes and contingency plans

	Event	Strategy
1	Random navigation and control errors	Design for 3-σ reliability
2	Loss of navigation en route to transfer vehicle	Switch to backup system(s) or track from surface (Earth) or track from cycler
3	Loss of navigation while docking	"Eyeball" it [2]
4	Failure of first upper stage	Retry after one orbit (if reparable) or abort to surface
5	Failure of second upper stage	Abort to surface
6	Second upper stage cuts off prematurely	None
7	Failure of rendezvous engine	Use remaining engines or switch to upper-stage engine
8	Loss of rendezvous propellant	Carry backup propellant in upper-stage engine[a]
9	Failure of docking engine or loss of propellant	Carry backup system or use transfer vehicle to dock

[a]May not be feasible without in situ propellant production.

propellant flow rate during thrusting and the inert mass of the dropped stages. Though necessary for an actual mission, we do not optimize the launch vehicle trajectory from any specific launch site. The time of the rendezvous burn is constrained to be 30 min before the two vehicles (nearly) collide, so the crew has time to switch to backup systems should the rendezvous engines fail. The option of departing orbit on a second try requires the simultaneous optimization of two trajectories (with departure times approximately one orbit period apart) that both meet the docking corridor constraints. A minimax optimization usually results in the same ΔV for both trajectories. This optimization defines the nominal trajectory.

Once in orbit, a second optimization is required to recalculate the first, second, and rendezvous-stage burns from the current estimated parking orbit (which is not optimal due to injection errors). The duration and direction for the departure and rendezvous burn provide 6 degrees of freedom to target the position and velocity state of the docking corridor. Once the taxi has departed orbit (either on the first or second try) the new time estimate for the closest approach to the cycler is calculated and the rendezvous burn is scheduled. Several hours pass between orbit departure and the rendezvous maneuver and docking to allow time for reoptimization based on the updated states. The direction and duration of the rendezvous burn minimize the ΔV from the docking velocity when the taxi is 10 km along the docking corridor. Again, the burn is calculated based on the current estimate of the taxi and cycler states. The next maneuver occurs at the estimated closest approach of the taxi to the docking corridor (which we define as the minimized product of the distance and angle from 10 km along the sun line). Of course, the taxi does not hit the exact corridor conditions because of navigation and thrust errors. To nullify these errors, the rendezvous engines burn periodically toward the docking corridor until the taxi is within 10 km of the cycler.

The docking phase begins when the taxi is 10 km away from the cycler. The taxi approaches the cycler along its sun line so that the cycler is illuminated on the docking side. The procedure for approach is depicted in Fig. 2.

The guidance algorithm for docking is as follows. The taxi maneuvers so that its velocity is directed toward the next range point (10 km, 8 km, etc., along the docking axis) at the specified range rate (40 m/s, 10 m/s, etc.). The docking guidance law is thus

$$\mathbf{\Delta V} = \dot{r}\mathbf{\Delta \hat{R}} - \mathbf{V} \tag{1}$$

where $\dot{r}$ is the desired range rate, $\mathbf{\Delta \hat{R}}$ is a unit vector directed from the taxi to the next range point, and $\mathbf{V}$ is the current taxi velocity. This $\mathbf{\Delta V}$ is implemented by firing the thrusters for a burn time of

$$t_b = m[1 - \exp(-\Delta V / g I_{sp})] g I_{sp} / T \tag{2}$$

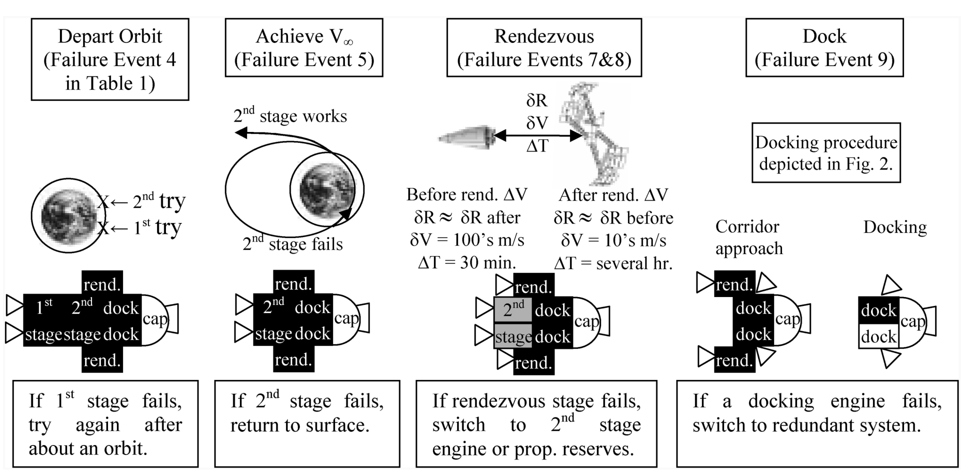

Fig. 1 Rendezvous profile and backup systems.

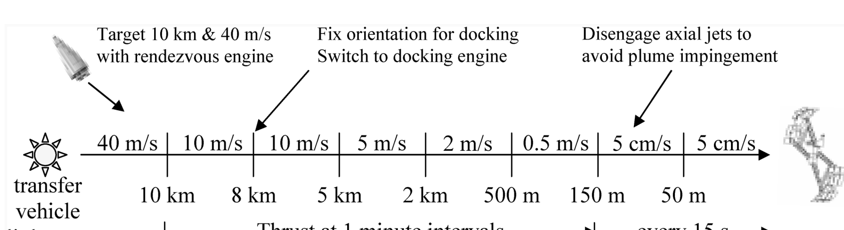

Fig. 2 Approach and docking profile.

for the rendezvous engine, which is directed along the desired $\mathbf{\Delta V}$, or for burn times of

$$\mathbf{t_b} = m[1 - \exp(-\mathbf{\Delta V}/gI_{sp})]gI_{sp}/T \tag{3}$$

for the docking thrusters, which are aligned orthogonally and burn for different times to achieve a given $\mathbf{\Delta V}$ array. In Eqs. (2) and (3) t_b is the burn time(s), m is the current taxi mass, g is the standard acceleration due to gravity at Earth's surface, I_{sp} is specific impulse, and T is the engine thrust. We note that this algorithm does not include gravity, which is in contrast to traditional rendezvous tactics [3–5]. Such a simple guidance law is possible because the spacecraft are sufficiently close to each other and far from the planet that the gravity perturbations are insignificant.

IV. Mission Specifications

The taxi mass is calculated to examine the effectiveness of our guidance strategy. We assume that the cycler vehicles follow the trajectories presented in [6] (which have lower average V_∞ than the Aldrin cycler [7]). These trajectories contain variations in V_∞ and the periapsis radius that are consistent with the flyby conditions of other cycler and semicycler trajectories. Because rendezvous occurs at planetary departure, only the outbound transfers are examined. We make the following assumptions to size the taxi:

1) The rendezvous time is 1 day.
2) The capsule mass is 6 t.
3) The heat shield mass fraction is 15%.
4) The circular orbit altitude is 200 km.
5) The Mars descent ΔV is 0.500 km/s to provide over 2 min of hover time.
6) The Mars ascent ΔV is 3.835 km/s to reach circular orbit from the surface (unspecified launch site).
7) The descent/ascent I_{sp} is 450 s.
8) The inert mass fraction $m_{inert}/m_{propellant}$ is 20 and 15% for the descent and ascent stages, respectively.

The other engine parameters are provided in Table 2. The 3-σ values for the position and velocity estimates are 300 m and 1 m/s for inertial navigation [12,13], and $0.03 \times R$ and 10 cm/s/km $\times R$ for relative navigation [14,15], where R is the distance between the taxi and the cycler. The minimum relative 3-σ values are 3 cm and 1 cm/s. The inertial navigation errors are used from orbit departure until the taxi reaches the docking corridor (at 10 km). The relative errors are used within the docking corridor (from 10 to 0 km). These uncertainty values derive from the performance of previous missions instead of the assumed capability of future engine and navigation hardware. The estimated state and applied thrust are the actual state and desired thrust plus random numbers. Though we do not model the launch vehicle trajectory, we set the circular orbit injection uncertainty to the inertial navigation uncertainty to provide an injection error.

V. Results

For each mission year over a 15-year Earth–Mars mission cycle (or seven synodic periods) we calculated 1000 rendezvous trajectories at Earth and Mars to provide a statistical base. Each trajectory included normal distributions on navigation and thrust uncertainties while following the rendezvous procedure of Sec. III. For comparison, we also provide values when the final taxi mass is maximized with none of the safety features in Table 2. The Mars taxi mass includes both descent and ascent stages, while the Earth taxi mass includes the stages to take the crew from low-Earth orbit (LEO) to the cycler. The taxi injected mass to LEO (IMLEO) value includes the Earth and Mars taxis plus a stage to send the Mars taxi to Mars via a minimum energy transfer. (This cargo stage has the same I_{sp} and inert mass fraction as the taxi upper stages.) The taxi IMLEO is the portion of injected mass to low-Earth orbit that is affected by the taxis. Additional mass to LEO could include cycler propellant and refurbishments, consumables, Mars payload, etc. The average taxi masses are provided in Table 3, and masses for each launch year are provided in [16].

We note that in Table 3 the taxi transfers with the additional safety features increased the IMLEO by about 40 t above the unsafe case, which provides the minimum mass estimate. When redundant rendezvous propellant is included the mass difference increases to about 65 t. Considering that the IMLEO for an entire mission to Mars can be at least 400 t, the additional mass seems worth the benefit. However a more detailed comparison of the costs and risks of cyclers versus other transportation architectures is necessary to decide how we shall travel to Mars. If we assume that propellant may be produced at Mars (e.g., by extracting water and creating liquid hydrogen and oxygen) then the IMLEO is significantly reduced. In this case the IMLEO is increased by 30 t above the unsafe case and redundant rendezvous propellant is available for both Earth and Mars rendezvous. Typical IMLEO values with in situ propellant production range from about 200 to 300 t.

The effect of each safety feature on the Earth taxi mass is explored in Table 4. In this table only one safety feature is incorporated at a time. (All six features from Table 4 are included to determine the "Safety measures in Table 1" mass values in Table 3.) We see that the largest increase in mass arises when redundant rendezvous propellant is carried on the taxi and the second largest is propellant for 3-σ reliability. The next largest contributor to the taxi mass is

Table 2 Engine parameters for rendezvous

Engine	Thrust, kN	I_{sp}, s	$m_{inert}/m_{propellant}$	Magnitude error (3σ), %	Direction error (3σ), deg
	200 × 2[a]				
First upper stage [8,9]	30.0 × 2	450	0.15	0.3	1.0
Second upper stage [8,9]	30.0 × 2	450	0.15	0.3	1.0
Rendezvous [10,11]	30.0 × 2	450	0.20	0.3	1.0
Docking [10,11]	4.0, 0.1[b]	300	0.25	1.0	0.3

[a]Two 200 kN engines are used at Earth and two 30 kN engines are used at Mars.
[b]The 100 N engine is used if the estimated burn time for the 4 kN engine is less than 0.1 s.

Table 3 Average taxi mass (in t) with and without redundant rendezvous propellant (RRP) and in situ propellant production (ISPP)

		Safety measures in Table 1			No safety measures		
RRP	ISPP	Mars taxi	Earth taxi	Taxi IMLEO	Mars taxi	Earth taxi	Taxi IMLEO
No	No	60.5	27.7	191.6	47.1	21.0	148.4
Yes	No	66.6	33.0	213.3	47.1	21.0	148.4
Yes	Yes	28.0	33.0	108.8	21.3	21.0	78.6

performing the rendezvous burn 30 min before the taxi reaches the cycler. The reason that this feature increases mass is that it reduces the rendezvous time, which causes an increase in rendezvous ΔV [17,18]. The second stage engine increases the dry mass for the rendezvous maneuver, but provides a backup system that already worked once (at departure). Including an entirely redundant docking propulsion system (both propellant and engines) does not increase the mass significantly because the ΔV is small compared with the other stages. Incorporating a "second chance" to depart LEO increases the ΔV because the taxi does not leave at the optimal time. The first chance to depart LEO occurs about half an orbit period before the optimal departure time and the second chance occurs about half an orbit period after the optimal time.

The taxi mass for any combination of these safety measures may be approximated by multiplying the product of the mass ratios with the nominal mass. For example, the taxi mass with both redundant docking propulsion and keeping the second stage tank is (approximately) $1.02 \times 1.06 \times 21.6$ t $= 23.4$ t, or including all of the safety features gives a ratio of 1.51 and a taxi mass of 32.6 t, which is close to the 33.0 t found in Table 3.

The standard deviation of the docking position error ellipse is 2.9 cm as given in Table 5 (where we assume normal distributions as shown in [16]). These values are typical for all launch years using the model discussed in Sec. III. Thus a docking radius of 10 cm is sufficient for 98.9% docking reliability. If the taxi approaches the cycler outside of the 10 cm docking radius, the crew still survives because the taxi can simply retry docking (as a proximity operation) until the two vehicles mate. The taxi intercepts the cycler at several cm/s, which is acceptable for current docking mechanisms [15]. The rendezvous time is typically a little longer than the desired 24 h because the taxi is usually more than the expected 10 km away from the cycler when the docking phase begins, and must therefore travel a farther distance (at the same controlled speed) to dock with the cycler. The largest standard deviation in ΔV belongs to the rendezvous stage, which performs a significant maneuver far from the planet and transfer vehicle. Proportionally, the docking standard deviation is largest when compared to its mean. Here, the errors contribute about as much ΔV as the expected value to effect a safe docking. The sensitivity of ΔV and docking radius to the different error sources is examined in Table 6. The position uncertainty affects ΔV the most and the velocity uncertainty has the greatest impact on the docking radius. The thrust errors (at these levels) may be considered insignificant when compared with the navigation uncertainties.

Table 4 Effect of safety measures on taxi mass for the Earth crew transfer during the 2009 mission

Failure event	Safety measure	Taxi mass, t	Mass ratio
Table 1	None	21.6	1.00
1	3σ	24.1	1.12
4	Second chance to depart LEO	21.7	1.01
7	Carry second stage tank	22.8	1.06
8	Carry redundant rendezvous propellant	24.8	1.15
7 and 8	Rendezvous 30 min early	23.0	1.07
9	Redundant docking propulsion	22.0	1.02

Table 5 Typical values for rendezvous parameters

Parameter	Mean	Standard deviation
Docking position	0 cm	2.9 cm
Docking velocity	−3.9 cm/s	0.8 cm/s
Transverse velocity	1.7 cm/s	1.3 cm/s
Rendezvous time	24.3 h	12 min
First upper-stage ΔV	1–3 km/s	3 m/s
Second upper-stage ΔV	1–2 km/s	2 m/s
Rendezvous-stage ΔV	200–700 m/s	40 m/s
Docking stage ΔV	55 m/s	14 m/s

VI. Conclusions

A relatively simple guidance algorithm may be used to dock the taxi with the cycler vehicle during hyperbolic rendezvous. This algorithm tracks the desired approach profile to allow the taxi to dock within 10 cm of the approach axis at a speed of 7 cm/s, 99% of the time, which is consistent with current docking capabilities. If the taxi approaches the cycler outside of the docking radius, then the docking procedure can be redone (with the backup docking system). The greatest danger to the crew is the second stage burning out prematurely or the taxi running out of propellant after the rendezvous burn. Safety measures and redundant systems are included in our rendezvous strategy to mitigate these problems at the expense of extra taxi mass. The increase in injected mass to low-Earth orbit is around 10–15% for a round-trip mission to Mars. We believe these measures provide an adequate safety margin for hyperbolic rendezvous, overcoming a major challenge of cycler and semicycler architectures. Thus these architectures become even more attractive for the sustained human exploration of Mars.

Acknowledgements

D. F. Landau's work has been sponsored by a National Science Foundation Graduate Research Fellowship. We thank Jonah Skoog for creating the vehicle images in Figs. 1 and 2.

References

[1] Young, K. A., and Alexander, J. D., "Apollo Lunar Rendezvous," *Journal of Spacecraft and Rockets*, Vol. 7, No. 9, Sept. 1970, pp. 1083–1086.

[2] Aldrin, E. E., "Line-of-Sight Guidance Techniques for Manned Orbital Rendezvous," Ph.D. Dissertation, Massachusetts Institute of Technology, Cambridge, MA, Jan. 1963.

[3] Pearson, D. J., "The Glideslope Approach," AAS Paper 19-162, 1989.

[4] Carter, T. E., "Closed-Form Solution of an Idealized, Optimal, Highly Eccentric Hyperbolic Rendezvous Problem," *Dynamics and Control*, Vol. 6, No. 3, July 1996, pp. 293–307.

[5] Goodman, J. L., "History of Space Shuttle Rendezvous and Proximity Operations," *Journal of Spacecraft and Rockets*, Vol. 43, No. 5, 2006, pp. 944–959.

[6] McConaghy, T. T., Landau, D. F., Yam, C. H., and Longuski, J. M., "Notable Two-Synodic-Period Earth-Mars Cycler," *Journal of Spacecraft and Rockets*, Vol. 43, No. 2, 2006, pp. 456–465.

[7] Byrnes, D. V., Longuski, J. M., and Aldrin, B., "Cycler Orbit Between Earth and Mars," *Journal of Spacecraft and Rockets*, Vol. 30, No. 3, May–June 1993, pp. 334–336.

Table 6 Sensitivity of ΔV and docking radius to thruster and navigation errors

	Rendezvous ΔV, m/s		Docking ΔV, m/s		Docking position, cm
Case	Mean	Standard deviation	Mean	Standard deviation	Standard deviation
Nominal	669	38	54	14	2.72
3 × thrust magnitude error	683	44	54	14	2.72
3 × thrust direction error	700	46	54	14	2.72
3 × navigation position error	711	66	89	20	4.03
3 × navigation velocity error	688	40	74	14	8.87

[8] Longuski, J. M., "Injection Module DV Accuracies for Galileo and International Solar Polar Missions," Jet Propulsion Laboratory Memorandum 343-733, 31 July 1982.
[9] Kangas, J. A., Potts, C. L., and Raofi, B., "Mars Exploration Rovers Launch Performance and TCM-1 Maneuver Design," AIAA Paper 2004-4984, 2004.
[10] Longuski, J. M., "Galileo Maneuver Analysis," AAS Paper 81-137, 1981.
[11] Lee, A. Y., and Hanover, G., "Cassini Spacecraft Attitude Control System Flight Performance," AIAA Paper 2005-6269, 2005.
[12] Larson, W. J., and Wertz, J. R., *Space Mission Analysis and Design*, 3rd ed., Microcosm Press, El Segundo, CA, 1999, p. 475.
[13] D'Amario, L. D., "Navigation of the Galileo Spacecraft," *The Three Galileos: The Man, the Spacecraft, the Telescope; Proceedings of the Conference, Dordrecht, Netherlands*, Springer–Verlag, New York, 1997, pp. 115–143.
[14] Kachmar, P. M., Chu, W., and Polutchko, R. J., "Use of a Laser Navigation Sensor for Automatic Rendezvous," *Navigation*, Vol. 39, No. 1, 1992, pp. 51–78.
[15] Zimpfer, D., and Spehar, P., "STS-71 Shuttle/Mir GNC Mission Overview," AAS Paper 96-129, 1996.
[16] Landau, D. F., and Longuski, J. M., "Guidance Strategy for Hyperbolic Rendezvous," AIAA Paper 2006-6299, 21–24 Aug. 2006.
[17] Friedlander, A. L., Niehoff, J. C., Byrnes, D. V., and Longuski, J. M., "Circulating Transportation Orbits Between Earth and Mars," AIAA Paper 1986-2009, 1986.
[18] Penzo, P. A., and Nock, K. T., "Hyperbolic Rendezvous for Earth-Mars Cycler Missions," AAS Paper 02-162, 2002.